Date:

Lesson Title: 3.5 Linear Equations & Rate of Change

Objective:

- Construct linear equations and their graphs using input and output tables.
- Calculate the rate of change for equations in intercept form using input and output variables.
- Explore the relationships among tables, scatter plots, recursive routines, and equations.

IN: Solve:

A. $10(x - 3) - 18 = 2$

B. $3 - 2(y + 5) = 4$

C. $\frac{7(-6 - 9z)}{10} - 15 = 6$

Linear Equations and Rate of Change

In this lesson you will continue to develop your skills with equations, graphs, and tables of data by exploring the role that the value of b plays in the equation

$$y = a + bx$$

You have already studied the intercept form of a linear equation in several real-world situations. You have used the intercept form to relate calories to minutes spent exercising, floor heights to floor numbers, and distances to time. So, defining variables is an important part of writing equations. Depending on the context of an equation, its numbers take on different real-world meanings. Can you recall how these equations modeled each scenario?

Example A:

The table relates the approximate wind chills for different actual temperatures when the wind speed is 15 mi/h. Assume the wind chill is a linear relationship for temperatures between –5° and 35°.

Temperature (°F)	–5	0	5	10	15	20	25	30	35
Wind chill (°F)	–25.8	–19.4	–13			6.2		19	25.4

a. What are the input and output variables?

b. What is the change in temperature from one table entry to the next? What is the corresponding change in the wind chill?

c. Use calculator lists to write a recursive routine that generates the table values. What are the missing entries?

Example A solution:

a. The input variable is the actual air temperature in °F. The output variable is the temperature you feel as a result of the wind chill factor.

b. For every 5° increase in temperature, the wind chill increases 6.4°.

c. The recursive routine to complete the missing table values is {–5, –25.8} ENTER and {Ans(1) + 5, Ans(2) + 6.4} ENTER. The calculator screen displays the missing entries.

Wind Chill investigation

Remember the Group Norms?

Investigation Handout

Complete Steps 1-8 in your group.

In this investigation you'll use the relationship between temperature and wind chill to explore the concept of rate of change and its connections to tables, scatter plots, recursive routines, equations, and graphs.

The data in the table represent the approximate wind chill temperatures in degrees Fahrenheit for a wind speed of 20 mi/h. Use this data set to complete each task.

Temperature (°F)	Wind chill (°F)
−5	−28.540
0	−21.980
1	−20.668
2	−19.356
5	−15.420
15	−2.300
35	23.940

Step 1:

Define the input and output variables for this relationship.

Step 2:

Plot the points and describe the viewing window you used.

Step 3:

Write a recursive routine that gives the pairs of values listed in the table.

Step 4:

Copy the table. Complete the third and fourth columns of the table by recording the changes between consecutive input and output values. Then find the rate of change.

Input	Output	Change in input values	Change in output values	Rate of change
−5	−28.540			
0	−21.980	5	6.56	$\frac{+6.56}{+5} =$
1	−20.668	1	1.312	
2	−19.356		1.312	$\frac{+1.312}{+1} =$
5	−15.420	3		
15	−2.300		13.12	$\frac{+13.12}{+10} =$
35	23.940			

Step 5:

Use your routine to write a linear equation in intercept form that relates wind chill to temperature. Note that the starting value, −28.540, is not the y-intercept. How does the rule of the routine appear in your equation?

Step 6:

Graph the equation on the same set of axes as your scatter plot. Use the calculator table to check that your equation is correct. Does it make sense to draw a line through the points? Where does the y-intercept show up in your equation?

Step 7:

What do you notice about the values for rate of change listed in your table? How does the rate of change show up in your equation? In your graph?

Step 8:

Explain how to use the rate of change to find the actual temperature if the weather report indicates a wind chill of 9.5° with 20 mi/h winds.

Example B:

This table shows the temperature of the air outside an airplane at different altitudes.

Input	Output
Altitude (m)	**Temperature (°C)**
1000	7.7
1500	4.2
2200	–0.7
3000	–6.3
4700	–18.2
6000	–27.3

Example B:

a. Add three columns to the table, and record the change in input values, the change in output values, and the corresponding rate of change.

Input	Output
Altitude (m)	Temperature (°C)
1000	7.7
1500	4.2
2200	–0.7
3000	–6.3
4700	–18.2
6000	–27.3

Example B:

b. Use the table and a recursive routine to write a linear equation in intercept form $y = a + bx$.

Input	Output
Altitude (m)	Temperature (°C)
1000	7.7
1500	4.2
2200	–0.7
3000	–6.3
4700	–18.2
6000	–27.3

Example B:

c. What are the real-world meanings of the values for a and b in your equation?

Input	Output
Altitude (m)	Temperature (°C)
1000	7.7
1500	4.2
2200	–0.7
3000	–6.3
4700	–18.2
6000	–27.3

Example B solution:

a. Record the change in input values, change in output values, and rate of change in a table. Note the units of each value.

Input	Output			
Altitude (m)	Temperature (°C)	Change in input values (m)	Change in output values (°C)	Rate of change (°C/m)
1000	7.7			
1500	4.2	500	−3.5	$\frac{-3.5}{500} = -0.007$
2200	−0.7	700	−4.9	$\frac{-4.9}{700} = -0.007$
3000	−6.3	800	−5.6	$\frac{-5.6}{800} = -0.007$
4700	−18.2	1700	−11.9	$\frac{-11.9}{1700} = -0.007$
6000	−27.3	1300	−9.1	$\frac{-9.1}{1300} = -0.007$

Example B solution:

b. Note that the rate of change, or slope, is always –0.007, or $\frac{-7}{1000}$. You can also write the rate of change as $\frac{-0.7}{100}$, so this recursive routine models the relationship:

$\{1000, 7.7\}$ ENTER

$\{\text{Ans}(1) + 100, \text{Ans}(2) - 0.7\}$ ENTER

Working this routine backward, $\{\text{Ans}(1) - 100, \text{Ans}(2) - 0.7\}$, will eventually give the result $\{0, 14.7\}$. So the intercept form of the equation is $y = 14.7 - 0.007x$, where x represents the altitude in meters and y represents the air temperature in °C.

Note that the starting value of the recursive routine is not the same as the value of the y-intercept in the equation.

Example B solution:

c. The value of a, 14.7, is the temperature (in °C) of the air at sea level. The value of b indicates that the temperature drops 0.007°C for each meter that a plane climbs.

Summary:

Rate of Change is ....

Out:

Turn to your elbow partner and share their thoughts of what you learned about linear equations today.

Write down what you discussed.

End of Class - Have a Great Day!!!